YANG-MILLS THEORY AS THE MASSLESS LIMIT

OF THE GAUGE INVARIANT RENORMALIZABLE MASSIVE MODEL.

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Infrared properties of the Yang-Mills field remain rather obscure.

Perturbative scattering matrix does not exist in the Yang-Mills theory, and the only sensible objects are the correlation functions of the gauge invariant operators, or gauge invariant infrared regularized theory. Simple infrared regularization by introducing a mass for the vector field, breaks the gauge invariance of the theory and the limit $m \to 0$ does not exist. Restoring gauge invariance by means of Stuekelberg formalism we loose renormalizability, and the limit $m \to 0$ does not coincide with the massless model.On the other hand in the Higgs model the limit $m \to 0$ does exist, but produces a different theory, describing not only the massless vector field but also a scalar particle.

In fact even the quantization of nonabelian gauge fields beyond perturbation theory is an unsolved problem (Gribov ambuguity).

It was discussed in the papers (A.A.Slavnov, Theoretical and Mathematical Physics 154(2008)213, A.A.Slavnov, JHEP 08(2008)047, A.Quadri, A.A.Slavnov JHEP 07(2010)087.) that impossibility to select a unique Lorentz invariant gauge beyond the perturbation theory is not the intrinsic property of the Yang-Mills model, but is related to its particular formulation. Adding new excitations which decouple asymptotically it is possible to quantize nonabelian gauge models in a manifestly Lorentz invariant way both in perturbation theory and beyond it. The infrared regularization based on this formulation was also proposed (A.A.Slavnov, Theoretical and Mathematical Physics 175(2013) 447. In this talk I am going to discuss the simpler method of gauge invariant infrared regularization, applicable both in perturbation theory and beyond it.

Having that in mind we propose to use for the gauge invariant infrared regularization of the Yang-Mills theory the following Lagrangian

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\varphi)^*(D_\mu\varphi) - (D_\mu\chi)^*(D_\mu\chi)$$
$$-g^{-1}[(D_\mu\varphi)^* + \alpha(D_\mu\chi)^*](D_\mu\hat{m}) - g^{-1}(D_\mu\hat{m})^*[D_\mu\varphi + \alpha D_\mu\chi] +$$
$$(1-\alpha^2)\frac{m^2 A_\mu^2}{2} + i[(D_\mu b)^*(D_\mu e) - (D_\mu e)^*(D_\mu b)] \quad (1)$$

$$\hat{m} = \{0, m\} \quad (2)$$

, where m is a constant, having the dimension of mass. The scalar fields (φ, χ are commuting, e, b are anticommuting) are parametrized by the Hermitean components

$$\Phi = \left(\frac{i\Phi_1 + \Phi_2}{\sqrt{2}}, \frac{\Phi_0 - i\Phi_3}{\sqrt{2}}\right) \quad (3)$$

This Lagrangian may be obtained from the gauge invariant Lagrangian, describing the interaction of the complex scalar doublets with the Yang-Mills field by the shift

$$\varphi \to \varphi - g^{-1}\hat{m}; \quad \chi \to \chi + \alpha g^{-1}\hat{m} \tag{4}$$

Hence the Lagrangian (1) is invariant with respect to the "shifted"gauge transformations.

In particular the transformation of the field $\varphi_-^a = \frac{\varphi^a - \alpha\chi^a}{\sqrt{2}}$ is

$$\delta\varphi_-^a = m\frac{1+\alpha}{2}\eta^a + \frac{1+\alpha}{2}\frac{g}{2}\varepsilon^{abc}\varphi_-^b\eta^c + \frac{1+\alpha}{2}\frac{g}{2}\varphi_-^0\eta^a$$

This Lagrangian at $\alpha = 1$, which corresponds to removing infrared regularization, is also invariant with respect to the supersymmetry transformations

$$
\begin{aligned}
\delta\varphi_\alpha^-(x) &= 2i\epsilon b_\alpha(x) \\
\delta e_\alpha(x) &= \epsilon\varphi_\alpha^+(x) \\
\delta b(x) &= 0
\end{aligned}
\qquad (5)
$$

where ϵ is a constant anticommuting parameter.

This invariance plays a crucial role in the proof of the equivalence of the model described by the Lagrangian (1) to the standard Yang-Mills theory.It provides the unitarity of the scattering matrix in the subspace which includes only three dimensionally transversal components of the Yang-Mils field.

The field ϕ^a_- is shifted by the gauge transformation by an arbitrary function, therefore one can put $\phi^a_- = 0$. This gauge is algebraic, but Lorentz invariant. It may be used beyond perturbation theory as well. In the case under consideration the massive theory with $\alpha \neq 1$ is gauge invariant but not unitary. It may seem strange as usually the gauge invariance is a sufficient condition of unitarity, because one can pass freely from a renormalizable gauge to the unitary one, where the spectrum includes only physical excitations. In the present case there is no "unitary"gauge. Even in the gauge $\phi^a_- = 0$, there are unphysical excitations.

A canonical quantization in the gauge $\varphi_-^a = 0$ requires introduction of ultralocal ghosts. So the gauge fixing is introduced by adding to the action the term

$$s \int d^4x \bar{c}^a \varphi_-^a = \int d^4x (\lambda^a \varphi_-^a - \bar{c}^a M_{ab} c_b) \tag{6}$$

$$M_{ab} = \delta_{ab}(m + \frac{g}{2}\varphi_-^0) + \frac{g}{2}\varepsilon_{abc}\varphi_-^b \tag{7}$$

Imposing the gauge condition $\varphi_-^a = 0$ we break the invariance of the effective action with respect to the supersymmetry transformation (5).

However, as the transition from one gauge to the other one may be achieved by a gauge transformation, and in the gauge $\partial_i A_i = 0$ the effective action is invariant with respect to the supertransformation (5), in the gauge $\varphi_-^a = 0$ it also must be invariant with respect to some supertransformation.The corresponding gauge function is a solution of the equation

$$\int d^4x \lambda^a(x) \partial_i (A^\Omega)_i^a(x) = \int d^4x \lambda^a(x) \varphi_-^a(x) \tag{8}$$

To analyze possible asymptotic states it is sufficient to find a solution of eq. (8)at $g = 0$. It is

$$\eta^a(x) = \frac{\partial_i A_i(x) - \varphi_-^a(x)}{m} \tag{9}$$

$\eta^a(x)$ are the parameters of the gauge group.

The functions $\eta^a(x)$ change under the supersymmetry transformations:

$$\eta^a(x) \to \eta^a(x) - i\frac{\sqrt{2}\varepsilon b^a(x)}{m} \tag{10}$$

Therefore

$$\tilde{A}_\mu^a(x) \to \tilde{A}_\mu^a(x) + i\frac{\sqrt{2}\varepsilon b^a(x)}{m} \tag{11}$$

The invariance with respect to the supertransformation(5) and the BRST transformations corresponding to the gauge invariant classical action in the gauge $\varphi^a_- = 0$ allows to find the transformation which leaves invariant the free effective action in the gauge $\varphi^a_- = 0$:

For asymptotic theory the symmetry transformations are

$$\begin{aligned}
\delta A_\mu^a &= \partial_\mu b^a \mu^{-1} \epsilon \\
\delta \phi^a &= 0 \\
\delta \phi^0 &= -b^0 \epsilon \\
\delta e^a &= \partial_\mu A_\mu^a \mu^{-1} \\
\delta e^0 &= -\partial^2 \phi^0 \mu^{-2} \\
\delta b^a &= 0 \\
\delta b^0 &= 0.
\end{aligned} \tag{12}$$

This invariance generates a conserved charge Q and the asymptotic states may be chosen to satisfy the condition

$$\hat{Q}_0|\psi>_{as}=0 \tag{13}$$

We want to prove that the Lagrangian(1+6) really describes the infrared regularization of the Yang-Mills theory. That means for $\alpha \neq 1$ it corresponds to a massive gauge invariant theory and in the limit $\alpha = 1$ it describes the usual three dimensionally transversal excitations of the Yang-Mills field.

These symmetries are sufficient to prove the renormalizability of the theory and the unitarity of the corresponding scattering matrix for $\alpha = 1$ in the subspace, which contains only three dimensionally transversal components of the Yang-Mills field. Scattering matrix for $\alpha \neq 1$ acts in large space containing the excitations with the negative energy and is not unitary in the physical subspace.

The spectrum:

Ghost excitations: $\varphi_{\pm}, b, e$, longitudinal and temporal components of A^a_μ
Physical excitations: three dimensionally transversal components of the Yang-Mills field.

The supersymmetry of the effective action generates a conserved nilpotent charge Q. Physical states are separated by the condition

$$Q|\psi>_{ph} = 0 \tag{14}$$

the states separated by this condition describe only three dimensionally transversal components of the Yang-Mills field.

The ghost excitations decouple.

Therefore for $\alpha \neq 1$ we have a gauge invariant theory , which describes massive Yang-Mills quanta, but is not unitary in the physical subspace. For $\alpha = 1$ we have usual massless Yang-Mills theory. So we succeeded in construction of the gauge invariant and renormalizable infrared regularization of the Yang-Mills theory. Obviously this regularization may be used both in perturbation theory and beyond it.